Categorical Relativity: Addition of Relative Velocities is Associative

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Introduction

Categorical relativity employs a category consisting of objects representing massive bodies (observers) and morphisms (arrows) that represent the relative velocity of one body (target) with respect to another (source).

Category Theory

Deals with abstract mathematical structures and relationships between them. Categories appear in most branches of mathematics and have been employed in computer science and mathematical physics. Category theory presents a unified languange and set of concepts that cross the boundries between many diverse and initially distinct subjects. Categories were first introduced by Samuel Eilenberg and Saunders Mac Lane in 1945, in connection with algebraic topology.

Instead of focusing on objects possessing a specific structure, category theory emphasizes the morphisms the structure-preserving processes between these objects.

We don't need much from category theory to begin applying it to relativity!

Relative Velocity

Unlike Lorentz boosts, pairs of relative velocities may be composed (added) only if the source of the first velocity matches the target of the second. Composition of relative velocities is associative.

The categorical structure of relative velocities matches our common intuition about velocity and is equally applicable to Galilean Relativity.

Relativity

Minkowski introduced the concept of (binary) relative velocity v in 1908 in the context of the special Lorentz transformations.

Zbigniew Oziewicz with Darius Swierk (thesis) considered addition of relative velocity in 1988.

$$Q = \gamma(P + v/c) \tag{1}$$

for P, Q time-like 4-vectors (obsrvers) where $\gamma = -P \cdot Q$

Tamas Matolcsi (1993) Spacetime without Reference Frames

Daniel Gottlieb (1997) http://arxiv.org/abs/q-alg/9603024

Observers P and Q determine different Euclidean subspaces

$$\{u \in M | P \cdot u = 0\} \neq \{u' \in M | Q \cdot u' = 0\} \quad (2)$$

Inverse Velocity

$$v = Q/\gamma - P \quad (3)$$

$$v' = P/\gamma - Q \quad (4)$$

$$v \neq -v' \quad (5)$$

Lorentz Boost (Matolcsi)

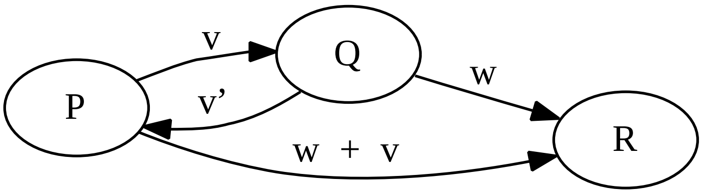
$$L(P, Q) \cdot v' = -v \quad (6)$$

$$L(P, Q) \cdot Q = P \quad (7)$$

Addition (Matolcsi, Oziewicz)

$$w = R/\gamma - Q \tag{8}$$

with $\gamma = -Q \cdot R$



$w + v =$

$$= \frac{(w - \frac{w \cdot v'}{v'^2} v)\sqrt{1 - v'^2} + (1 - \frac{w \cdot v}{v^2})v}{1 - w \cdot v'} \tag{9}$$

$$= \frac{\gamma v + w}{\gamma(1 - \frac{w \cdot v'}{c^2})} + \frac{w \cdot v'}{c(1 - \frac{w \cdot v'}{c^2})} P \tag{10}$$

All inner products are defined within Euclidean subspace. Hyperbolic geometry is not necessary (Oziewicz).